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# A Modified Redundancy Optimization of k-out-of-n: G System

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***ABSTRACT:***

*This paper discusses the optimum designing of K-out-of-n: G redundant system in the presence of 'intrinsic' failures defining a long run expected cost per unit time measure namely* ***System Average Cost Rate****. This research work develops a new modified refined optimum redundancy policy parallel to Nakagawa [2]. The present model provides a cost effective optimum redundancy (n*) as an improvement to Nakagawa [2] model under the influence of failure rate and time of operation which is obvious in Reliability analysis. The results are tabulated and graphs are presented for the model for comparison.*

***Key words****: Optimization, k-out-of-n: G redundant system, Exponential failure law.*

### 1. INTRODUCTION:

Redundancy often used to achieve a required reliability for a system with unreliable parts. A k-out-of-n structure is optimal among redundant structures under certain conditions. Several papers have treated the problem of optimum redundancy. Rau and Ben-Dov studied the problem of determining k maximizing the reliability for a given n. Nakagawa [2] studied the problems of determining optimal numbers of redundant units which minimize the mean cost rate. In these and other models, only random failures (intrinsic) of components result in system failure.

This paper discusses refined optimum policy of k-out-of-n redundant system parallel to Nakagawa [2] model minimizing System Average cost rate. The optimal number of redundant units is shown to be unique. This paper establishes the results that the present optimum redundancy is highly cost effective while retaining the improvement of Reliability compared to Nakagawa [2] model.

## 2. ASSUMPTIONS:

1. The components in the system are subjected to fail independently & randomly.
2. Failures times of all components obey exponential probability law.
3. Individual failures occur at a constant rate.
4. Replacement times are negligible.
5. The planning horizon is infinite.

## 3. NOTATION:

| | |
|---|---|
| $c_1$ | : Acquisition cost of one unit |
| $c_2$ | : Additional cost of the system which is replaced at failure. |
| $c_2^1$ | : $c_2 / c_1$ |
| n | : Number of redundant units |
| * | : Implies the optimal value |
| k | : Minimum number of components required for successful operation of the system |
| t | : Time factor ( In units of time Min/Hrs/ Days) |
| p | : Module (component) reliability (q = 1- p) |
| $\lambda$ | : Constant failure rate of each component. (No. of failures per unit time) |
| $n^*_M$ | : Optimal redundancy for proposed Modified model |
| $n^*_N$ | : Optimal redundancy for Nakagawa model |
| MTTF | : Mean time to failure of the system |
| $S_n$ | : Mean time to failure (MTTF) of the system |
| R (n,k,t) | : Reliability of the system |
| C (n,$\lambda$, t) | : System average cost rate. |

## 4. MODEL:

The k-out-of-n: G redundant system where minimum of k units should work for the successful operation of system where n is the redundant number of components and k < n. The components in the systems are subject to fail individually with failure rate $\lambda$.

In this section alternative optimum redundancy is developed using the cost function. i.e., System average cost rate unlike Nakagawa [2].

The System average cost rate is defined as

$C(n, \lambda, t) = E(C) / E(Y)$

where, system average cost is

$$E(C) = c_1 n + c_2 (1 - R_s(k,n,t)) \quad \text{(see Pham [5] )} \qquad (1)$$

$$\text{and} \quad R_s(k,n,t) = \sum_{i=k}^{n} \binom{n}{i} (e^{-\lambda t})^i (1 - e^{-\lambda t})^{n-i} \qquad (2)$$

and E(Y) = mean time to system failure (MTTF),

$$S_n = \int_0^{\infty} R_s(k,n,t)\, dt = (1/\lambda) \sum_{i=k}^{n} (1/i) \qquad (3)$$

Therefore the System average cost rate can be expressed as

$$C(n, \lambda, t) = (c_1 n + c_2(1 - R_s(k,n,t))) / S_n \quad \text{(see Nakagawa [2])} \qquad (4)$$

Further, $S_n$ obeys,

(i) $(S_{n+1} - S_n)$ decreases in n, and $\lim_{n \to \infty} \{S_{n+1} - S_n\} = 0$

(ii) $S_{n+1} = 1 / (\lambda (n+1)) + S_n$.

A necessary condition for $n^*_M$ to minimize C (n, $\lambda$, t) as given in (4) is

$$C(n+1) \geq C(n) \text{ and } \quad C(n) < C(n-1) \qquad (5)$$

i.e., $C(n+1) - C(n) \geq 0$

$\{\{[c_1 (n+1) + c_2 (1- R(k,n+1,t))] / S_{n+1}\} - \{ [c_1 (n) + c_2 (1- R(k,n,t))] / S_n \}\} \geq 0$

Further it reduces to

$$c_1 S_n - n c_1 ( S_{n+1} - S_n) \geq c_2\{( S_{n+1} - S_n ) - (R(k,n,t) S_{n+1} - S_n R(k,n+1,t) )\} \qquad (6)$$

The above equation can be expressed as

$$c_1 (1/\lambda_1) \sum_{i=k}^{n} (1/ i) - n c_1 (1/\lambda_1) \left( \sum_{i=k}^{n+1} (1/ i) - \sum_{i=k}^{n} (1/ i)\right) \geq$$

$$c_2\left\{\left((1/\lambda_1)\left(\sum_{i=k}^{n+1} (1/ i) - \sum_{i=k}^{n} (1/ i)\right)\right) - \left(R(k,n,t) (1/\lambda_1) \sum_{i=k}^{n+1} (1/ i) - R(k,n+1,t) (1/\lambda_1)\sum_{i=k}^{n} (1/ i)\right)\right\}$$

$$\Rightarrow c_1 \sum_{i=k}^{n} (1/ i) - n c_1 (1/(n+1)) \geq$$

$$c_2 \left\{(1/(n+1)) - \left(R(k,n,t) \sum_{i=k}^{n+1} (1/ i) - R(k,n+1,t) \sum_{i=k}^{n} (1/ i)\right)\right\}$$

Therefore,

$$\left( \sum_{i=k}^{n+1}(1/i) -1\right)/\left\{(1/(n+1))(1- (R(k,n,t)) + \sum_{i=k}^{n}(1/i) (R(k,n+1,t) - R(k,n,t))\right\} \geq c_2 /c_1$$

Therefore, $Z (n) \geq c_2 / c_1 = c_2^1$ (7)

where, $Z(n) =\left(\sum_{i=k}^{n+1}(1/i) -1\right)/\left\{(1/(n+1))(1-(R(k,n,t)) + \sum_{i=k}^{n}(1/i) ( R(k,n+1,t) -R(k,n,t))\right\}$

Thus optimal redundancy $n^*_M$ depends on k, $c_2^1$

Thus (5) is equivalent to $Z (n) \geq c_2^1$ and $Z (n - 1) < c_2^1$ (8)

From the properties (i) and (ii) $[Z (n+1) - Z (n)] > 0$,

Hence Z (n) is increasing in n, and $\lim_{n\to\infty} Z (n) \to\infty$

Therefore, if $Z (k) < c_2^1$, the optimal redundancy will be $n^*_M$ which is the smallest n such that $Z (n) \geq c_2^1$, otherwise, $n^*_M = k$.

## 5. EXAMPLE AND DISCUSSION:

To illustrate the results of optimum redundancy of the above model in detail, an example is considered and the results were derived for various values of failure rates $\lambda$ = 0.001, 0.01, 0.05 and 0.5 and time t = 2, 15, 30 with cost ratio $c_2^1$ (= $c_2 / c_1$) = 50, 100, 1000.

The detailed sketch of values is presented in tables 5.1 to 5.3. For further illustration of results graphs were plotted for failure rate ($\lambda$) **v/s** optimal redundancy (n*) and seen in figures 5.1 to 5.3. It is observed from the figures that as the mission time and the value of k increases the optimal redundancy (n*) also increase (see column five of tables 5.1 to 5.3 and figures 5.1 to 5.3). Further it is interesting to note that the modified model suggests a cost effective redundancy when compared to Nakagawa model [2].

In modified model it is observed that when failure rate is more the model suggests more number of redundant units compared to smaller units of failure rate (see columns 5 of tables 5.1, 5.2, 5.3 and figures 5.1 to 5.3). This seems to be more realistic, unlike the Nakagawa model [2]. For various values of failure rate the Nakagawa model [2] suggests same optimal redundant units (see column 4 of tables 5.1, 5.2, 5.3 and figures 5.1 to 5.3).

Naturally the failure rate and time have influence on the system such a way that as the failure rate and time increase, the redundancy also increases. But in case of Nakagawa [2] model the failure rate and operational time (t) have no influence on redundancy (see lower and upper rows of tables 5.1 to 5.3 and figures 5.1 to 5.3). However in the proposed model there is a change in redundancy for change of failure rate (λ) and mission time (t). Thus the proposed modified model is an improved and modified version when compared to Nakagawa model [2] in terms of real applications.

**Table 5.1: Optimum values of redundancy for failure rate λ = 0.01 and with variation of time.**

| **Time units operational (t)** | **k** | $c_2^1 = c_2 / c_1$ | **Optimal redundancy** | |
|---|---|---|---|---|
| | | | **Nakagawa model ($n^*_N$)** | **Modified model ($n^*_M$)** |
| 2 | 3 | 50 | 32 | 6 |
| | | 100 | 50 | 6 |
| | | 1000 | 271 | 6 |
| | 4 | 50 | 36 | 9 |
| | | 100 | 56 | 9 |
| | | 1000 | 292 | 9 |
| | 5 | 50 | 40 | 11 |
| | | 100 | 61 | 11 |
| | | 1000 | 309 | 11 |
| 15 | 3 | 50 | 32 | 7 |
| | | 100 | 50 | 7 |
| | | 1000 | 271 | 8 |
| | 4 | 50 | 36 | 9 |
| | | 100 | 56 | 9 |
| | | 1000 | 292 | 10 |
| | 5 | 50 | 40 | 11 |
| | | 100 | 61 | 11 |
| | | 1000 | 309 | 12 |
| 30 | 3 | 50 | 32 | 8 |
| | | 100 | 50 | 9 |
| | | 1000 | 271 | 11 |
| | 4 | 50 | 36 | 10 |
| | | 100 | 56 | 11 |
| | | 1000 | 292 | 13 |
| | 5 | 50 | 40 | 13 |
| | | 100 | 61 | 13 |
| | | 1000 | 309 | 15 |

**Table 5.2: Optimum values of redundancy for failure rate λ = 0.05 and with variation of time.**

| Time units operational (t) | k | $c_2^1 = c_2 / c_1$ | Optimal redundancy | |
|---|---|---|---|---|
| | | | Nakagawa model ($n^*_N$) | Modified model ($n^*_M$) |
| 2 | 3 | 50 | 32 | 6 |
| | | 100 | 50 | 7 |
| | | 1000 | 271 | 7 |
| | 4 | 50 | 36 | 9 |
| | | 100 | 56 | 9 |
| | | 1000 | 292 | 9 |
| | 5 | 50 | 40 | 11 |
| | | 100 | 61 | 11 |
| | | 1000 | 309 | 11 |
| 15 | 3 | 50 | 32 | 13 |
| | | 100 | 50 | 15 |
| | | 1000 | 271 | 19 |
| | 4 | 50 | 36 | 16 |
| | | 100 | 56 | 18 |
| | | 1000 | 292 | 22 |
| | 5 | 50 | 40 | 19 |
| | | 100 | 61 | 21 |
| | | 1000 | 309 | 26 |
| 30 | 3 | 50 | 32 | 25 |
| | | 100 | 50 | 29 |
| | | 1000 | 271 | 40 |
| | 4 | 50 | 36 | 31 |
| | | 100 | 56 | 35 |
| | | 1000 | 292 | 48 |
| | 5 | 50 | 40 | 37 |
| | | 100 | 61 | 41 |
| | | 1000 | 309 | 54 |

**Table 5.3: Optimum values of redundancy for failure rate λ = 0.5 and with variation of time.**

| Time units operational (t) | k | $c_2^1 = c_2 / c_1$ | Optimal redundancy | |
|---|---|---|---|---|
| | | | Nakagawa model ($n^*_N$) | Modified model ($n^*_M$) |
| 2 | 3 | 50 | 32 | 17 |
| | | 100 | 50 | 19 |
| | | 1000 | 271 | 25 |
| | 4 | 50 | 36 | 21 |
| | | 100 | 56 | 23 |
| | | 1000 | 292 | 29 |
| | 5 | 50 | 40 | 24 |
| | | 100 | 61 | 26 |
| | | 1000 | 309 | 33 |
| 15 | 3 | 50 | 32 | 32 |
| | | 100 | 50 | 50 |
| | | 1000 | 271 | 271 |
| | 4 | 50 | 36 | 36 |
| | | 100 | 56 | 56 |
| | | 1000 | 292 | 292 |
| | 5 | 50 | 40 | 40 |
| | | 100 | 61 | 61 |
| | | 1000 | 309 | 309 |
| 30 | 3 | 50 | 32 | 32 |
| | | 100 | 50 | 50 |
| | | 1000 | 271 | 271 |
| | 4 | 50 | 36 | 36 |
| | | 100 | 56 | 56 |
| | | 1000 | 292 | 292 |
| | 5 | 50 | 40 | 40 |
| | | 100 | 61 | 61 |
| | | 1000 | 309 | 309 |

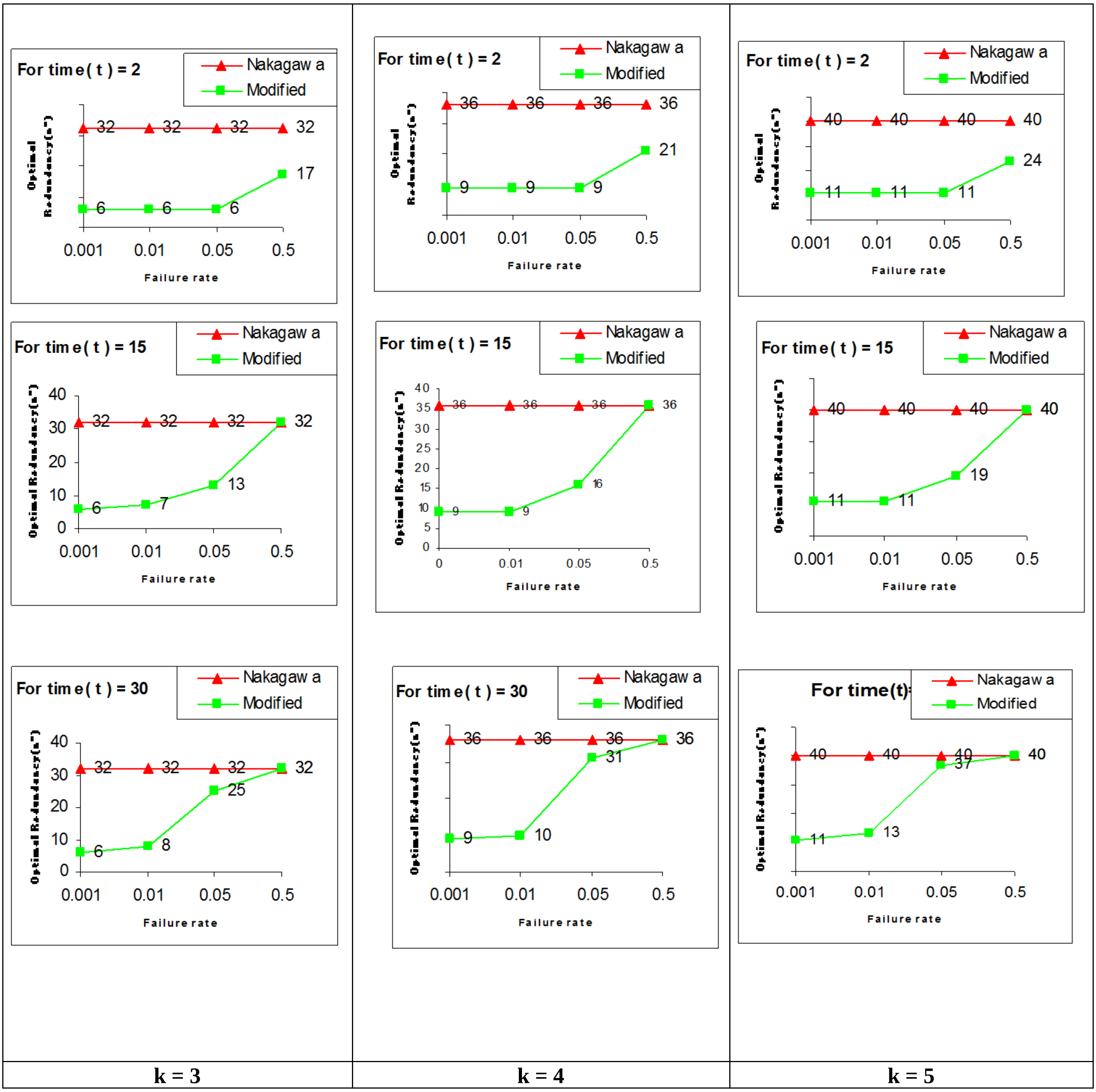


**Fig. 5.1: Optimal Redundancy (n*) v/s Failure rate (λ) for $c_2^1 = 50$ and t = 2, 15, 30**

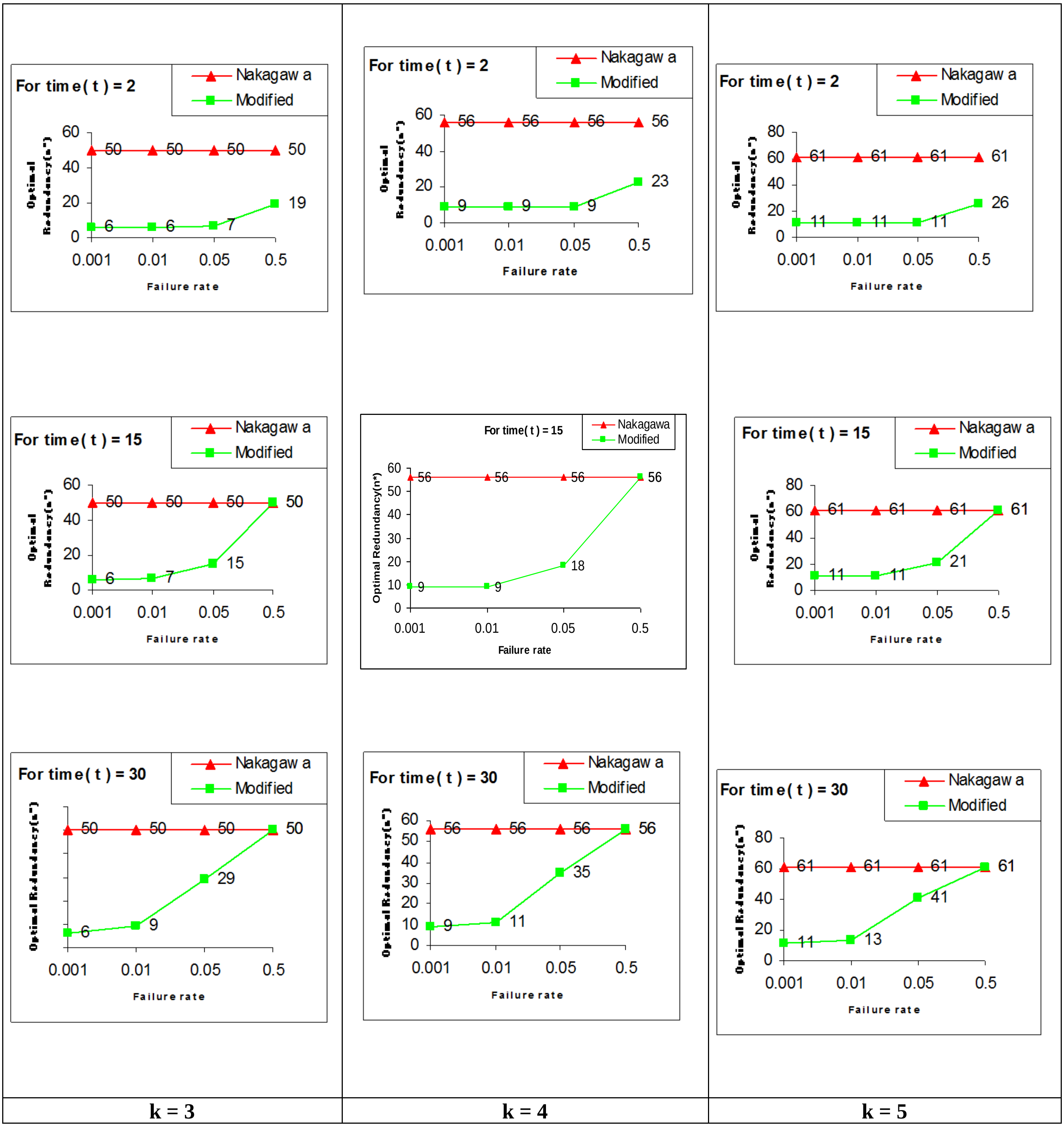


**Fig. 5.2: Optimal Redundancy (n*) v/s Failure rate (λ) for $c_2^1$ = 100 and t = 2, 15, 30**

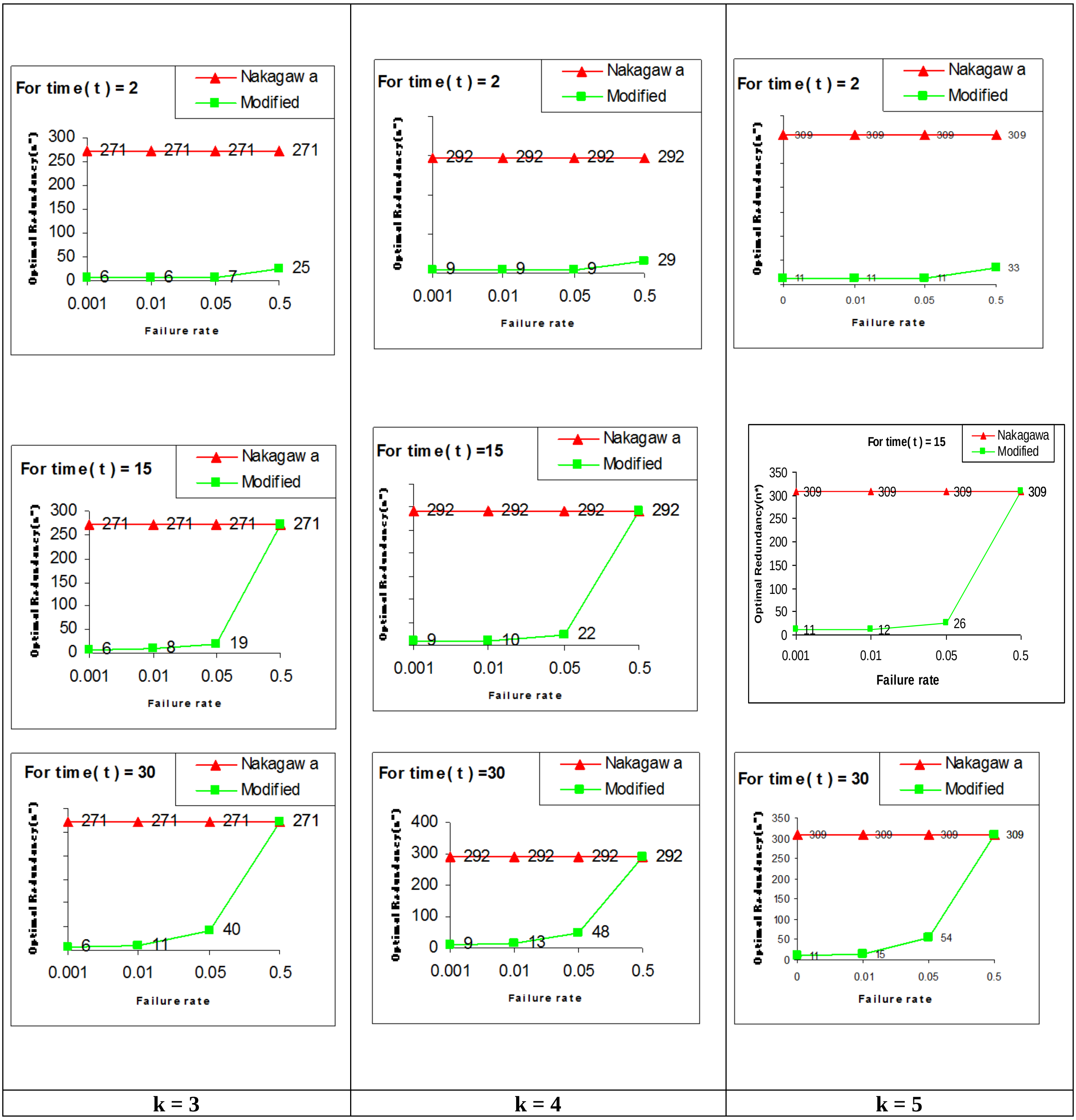


**Fig. 5.3: Optimal Redundancy (n*) v/s Failure rate (λ) for $c_2^1 = 1000$ and t = 2, 15, 30**

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